Portfolio Optimization under Regulation T: A Comparative Study of the Markowitz Model and the Index Model

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Abstract. The traditional risk management system of Modern portfolio theory faces operational barriers because investment regulations restrict both borrowing and selling short. The existing regulatory boundaries which limit leverage and short-selling operations create difficulties for standard optimization methods while making alternative portfolio models less stable and harder to implement. The research investigates how Regulation T gross-exposure limits affect the performance of Markowitz Model and Single-Index Model portfolios. The study evaluates operational performance and model stability between these two models by using U.S. equity data and market index returns and risk-free proxy to create efficient frontiers and global minimum variance portfolios and tangency portfolios. The Index Model produces efficient frontiers that remain stable while delivering higher risk-adjusted returns at reduced risk levels than the Markowitz Model. The Markowitz Model generates frontiers with increased noise and needs more computational resources which makes it more vulnerable to estimation errors. The Markowitz approach provides superior theoretical completeness but its practical use becomes less effective when leverage restrictions exist while the Index Model shows better compliance with regulatory needs. The research proves that spreadsheet optimization tools encounter operational barriers which need sophisticated computational systems to deliver enhanced performance and accuracy. The research proves that portfolio models need to fulfill both theoretical requirements and current regulatory standards to achieve their best performance. Investors and researchers obtain superior outcomes by using methods which merge mathematical accuracy with operational ease to link theoretical frameworks with actual portfolio construction.

Keywords: Regulation T, Index Model, Markowitz Model.

1. Introduction

During the 1950s, the mean–variance framework established by Markowitz created a systematic method to manage risk and return, which became the basis for contemporary portfolio theory. The expansion of the field of financial economics has used portfolio optimization as its core subject since Harry Markowitz introduced his groundbreaking work, and growing datasets created difficulties for traditional optimization methods because they struggled with estimation errors, computational complexity, and input sensitivity. The current financial market environment has seen the development of new optimization techniques, which include index-based models, robust covariance methods, and machine learning algorithms to overcome existing limitations. The implementation of traditional optimization models becomes more complicated because Regulation T restricts leverage and short-selling activities, which forces models to work within actual regulatory restrictions.

This research evaluates the Full Markowitz Model (MM) and the Single-Index Model (IM) through a comparative analysis between these two established optimization methods. The Full Markowitz Model provides complete risk assessment through its full covariance matrix, but its practical use becomes unstable when dealing with numerous assets. The Single-Index Model achieves better estimation robustness through its use of one market factor to explain correlations, yet it sacrifices some of its general applicability. The research evaluates both models under Regulation T gross-exposure constraints using 21 U.S. equities and the S&P 500 index and the 1-month Fed Funds rate as a risk-free proxy. The research uses Excel Solver with SolverTable to generate efficient frontiers and global minimum variance (GMV) portfolios, tangency portfolios, and capital allocation lines (CAL) for model comparison.

The research aims to assess the operational efficiency, stability, and real-world applicability of MM and IM when subject to regulatory leverage restrictions. The research uses empirical model comparison to enhance knowledge about how different estimation methods impact portfolio results and how regulatory rules affect available investment choices. The research establishes a connection between theoretical portfolio models and practical implementation to provide useful knowledge for institutional investors, academic researchers, and policy makers who need effective portfolio solutions for real-world scenarios.

2. Literature Review

The Markowitz mean-variance paradigm functions as the fundamental basis of portfolio theory although its performance in real-world applications remains problematic because of estimation errors and unstable large datasets. Boyd et al. review seventy years of portfolio theory development through a historical analysis which shows both the theoretical beauty and the practical implementation barriers of the model for real-world market applications [1]. The need for research exists to develop constraints and covariance estimation methods and alternative models which enhance robustness. Research studies examine how portfolio constraints affect investment decisions. The authors Jagannathan and Ma demonstrate that implementing constraints which seem limiting actually leads to better out-of-sample results because they reduce the impact of parameter estimation errors [2]. Wu presents empirical evidence which demonstrates that the Single Index Model generates more stable weights and smoother efficient frontiers than the Markowitz Model when trading under realistic market conditions [3]. The research supports the dual function of constraints because they fulfill regulatory needs and create optimization stability. The research stream dedicated to covariance estimation improvement has received significant attention. The well-conditioned covariance estimator developed by Ledoit and Wolf delivers superior results when dealing with large-dimensional systems [4]. The authors Zhao, Ledoit and Jiang demonstrate that risk reduction becomes possible through the combination of leverage limits with gross-exposure constraints when using shrinkage methods [5]. The nonlinear shrinkage method developed by Ledoit and Wolf produces optimized covariance matrices which generate better optimization results [6]. The research by DeMiguel, Garlappi and Uppal shows that the basic 1/N diversification method delivers better results than mean-variance portfolios when estimation errors become significant [7]. The research by Dutta and Jain demonstrates that shrinkage methods outperform precision-based methods when used for covariance estimation in practical applications [8]. The ongoing discussion between theoretical optimality and empirical robustness continues to be a central theme in this research area. The research now applies these findings to actual investment scenarios. Lolic develops enhanced methods for managing multi-asset class portfolios which minimize risk concentration and trading expenses while maintaining mean-variance optimization [9]. Zhang and his co-authors develop a distributionally robust mean-variance approach which uses robust statistics to build portfolios for high-dimensional financial markets [10]. The research demonstrates how portfolio theory has progressed from its basic principles into models which combine empirical strength with regulatory compliance and computational efficiency. The research demonstrates that portfolio construction needs to minimize estimation errors while maintaining the theoretical benefits of mean-variance optimization through various approaches including constraints and shrinkage and index models and robust optimization.

3. Methodology

When receiving the paper, we assume that the corresponding authors grant us the copyright to use the paper for the book or journal in question. When receiving the paper, we assume that the corresponding authors grant us the copyright to use. The research implements two separate models. The Full Markowitz Model (MM) is the first model that requires a complete covariance matrix for its operation. The process of calculating n expected returns, n variances, and n(n-1)/2 covariances is required for this method. The extensive number of required inputs makes MM computationally demanding and susceptible to errors during estimation. The Index Model (IM) represents the second model, which uses a simplified covariance structure based on market index correlation as the primary driver of asset relationships. The return of each asset follows the formula $r_i = \alpha_i + \beta_i r_M + \varepsilon_i$, where r_M stands for market return and β_i shows the market factor sensitivity of the asset, and ε_i represents the firm-specific residual. The variance decomposition formula shows that σ_p^2 equals $\beta_p^2 \sigma_M^2$ plus $\Sigma w_i^2 \sigma_{\varepsilon i}^2$ where β_p equals the sum of w_i times β_i. The IM shows better stability when handling unreliable data points according to previous research on robust optimization. The research by Jagannathan and Ma [2] demonstrates how constraints reduce estimation inaccuracies which results in more dependable frontiers. Wu [3] demonstrates through his research that the Index Model generates more stable and smooth results than the complete Markowitz model particularly when constraints are implemented. The research shows that IM frontiers generate more stable results than MM frontiers in actual market scenarios.

The main focus of this research study is the FINRA rule-based Regulation T constraint, which limits investor leverage through a requirement that investors must fund at least half of their account positions with equity. The constraint is expressed as $\Sigma |w_i| \leq 2$. The definition establishes maximum exposure limits that block unrealistic investment strategies that heavily depend on borrowed funds. Research evidence shows that portfolio stability depends heavily on the implementation of gross-exposure limits. The research by Jagannathan and Ma [2] shows that portfolio weight constraints minimize extreme positions and lead to better out-of-sample results while Zhao, Ledoit, and Jiang [5] prove that combining shrinkage with exposure limits produces better portfolio results. The Regulation T framework achieves better alignment between optimization results and actual investment scenarios through its restrictions on both excessive leverage and short selling activities.

The research dataset consists of 21 U.S. equities, which span different industries, together with the S&P 500 index (SPX) and the 1-month Fed Funds rate (FEDL01) as the risk-free rate proxy. The dataset spans twenty years from 2004 to 2024, which includes two significant market events: the 2008 financial crisis and the COVID-19 pandemic. The aggregation of daily returns into monthly intervals follows standard empirical finance recommendations to minimize microstructure effects and non-Gaussian noise. The use of longer-horizon returns according to Jagannathan and Ma [2] leads to better portfolio input estimation and Wu [3] shows that these adjustments enhance the stability of MM and IM frontiers in actual research. The optimization inputs, including expected returns, variances, covariances, and betas, use aggregated data to represent long-term market behavior instead of short-term market microstructure effects. The optimization process utilized Excel Solver together with SolverTable to produce multiple portfolio points that followed the efficient frontier. The analysis produced four key outputs from both MM and IM systems, which included the Efficient Frontier for maximum returns at different risk levels, the Global Minimum Variance (GMV) Portfolio for risk minimization, the Tangency Portfolio for Sharpe ratio maximization, and the Capital Allocation Line (CAL) that links the risk-free asset to the tangency portfolio. The empirical results obtained from these outputs enabled researchers to assess how MM and IM systems performed under the Regulation T constraint. Research shows that MM performance can be enhanced by using shrinkage estimators from Ledoit and Wolf [4, 6] and distributionally robust covariance methods developed by Zhang, Jing and Kao [10]. The techniques generate more reliable covariance estimates which result in superior risk–return performance in real-world usage.

4. Empirical Results

The study operate under Regulation T gross-exposure restrictions. It provides both efficient frontier diagrams and portfolio results for the global minimum variance (MinVar) and tangency portfolios. The frontiers for the IM empirical findings from this research demonstrate how the Index Model (IM) and Full Markowitz Model (MM model) appear in Figures 1 and 2, while Tables 1 and 2 show complete numerical data for portfolios derived from each model.

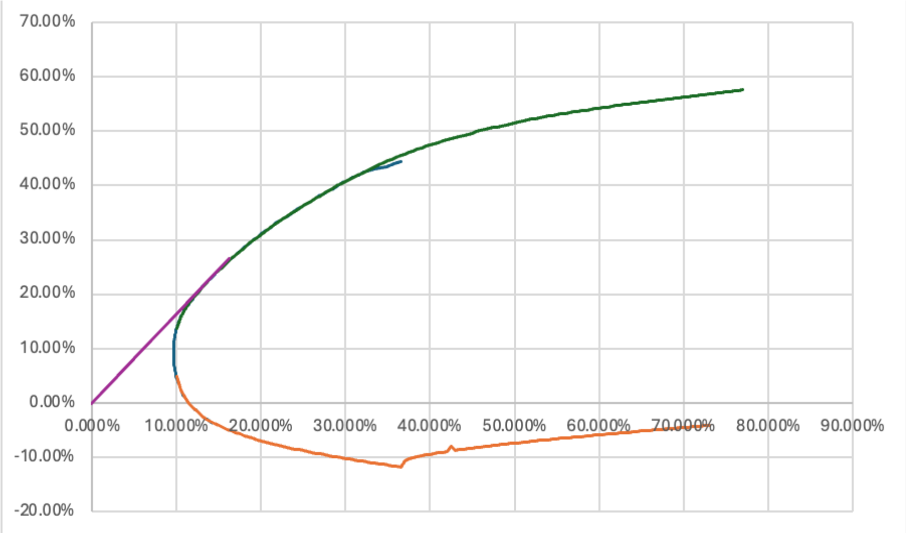

Figure 1. Efficient Frontier of the IM Model

The IM model operating under Regulation T generates an efficient frontier, which appears in Figure 1. The curve starts with negative returns when the standard deviation remains low because it shows possible portfolios that use extreme positions in low-risk assets, but result in losses after applying constraints. The curve shows a steady improvement as risk levels increase. The portfolio returns approach positive double-digit values when the risk level reaches approximately 30% standard deviation. The frontier shows a maximum expected return of 60% when the risk level reaches 70%. The Regulation T gross-exposure limit enables each point on the curve to represent portfolios that use 21 assets and S&P 500 index weights. The frontier starts with the minimum achievable return from constrained investments and ends with the maximum possible return at its most risky point.

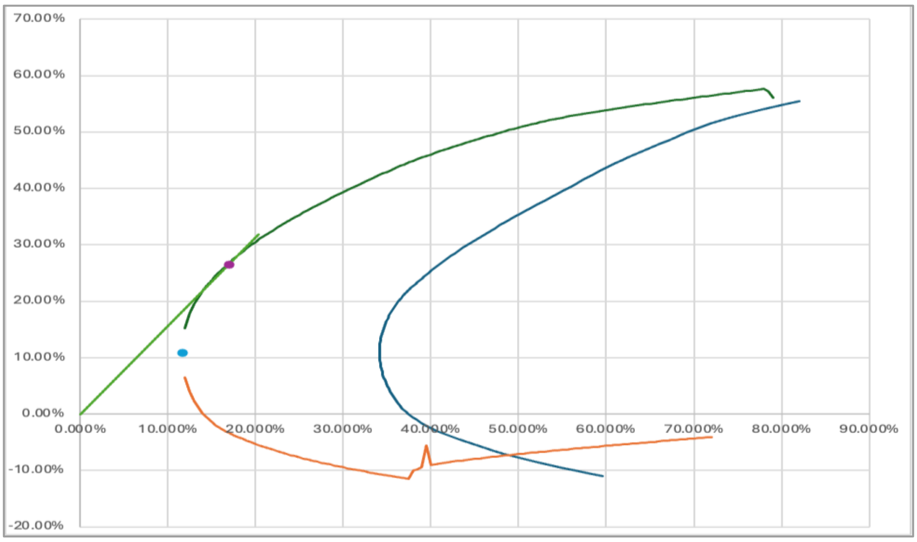

Figure 2. Efficient Frontier of the MM Model

The MM model generates its efficient frontier under the same constraint that appears in Figure 2. The MM curve starts at 0% return when the standard deviation reaches 0% because it represents either the risk-free asset or an insignificant risky asset allocation. The frontier shows a positive upward trend as risk increases until it reaches 10-20% returns at standard deviation levels between 20-40%. The curve maintains its upward trend until it approaches 60% expected return when the portfolio standard deviation reaches 80%. The optimization procedure under Regulation T generates each feasible combination of the 21 assets, which are plotted in the results. The standard deviation of returns serves as the horizontal axis, while the expected portfolio return functions as the vertical axis in this figure. The two figures present a visual representation of the feasible opportunity sets that result from implementing the IM and MM specifications.

Table 1. Portfolio Results for the MM Model

MM		SPX	AMZN	BKNG	GOOGL	NFLX	NKE	F
	MinVar	18.00%	-3.28%	-1.50%	0.64%	3.25%	0.11%	-0.48%
	MaxSharpe	**-21.37%**	**2.41%**	**19.87%**	**13.70%**	**11.39%**	**4.08%**	**-9.28%**
COST	**HD**	**LOW**	**MO**	**KO**	**PEP**	**AMGN**	**ABT**	**DHR**
9.92%	7.90%	-7.78%	10.13%	19.52%	21.18%	3.31%	6.12%	7.08%
31.70%	7.01%	0.62%	25.54%	-0.02%	5.44%	2.26%	1.76%	-0.43%
MDT	**TMO**	**UNH**	**BMY**	**CRM**	**AXP**	**returns**	**StDev**	**Sharpe**
1.40%	-2.82%	4.87%	6.93%	0.11%	-4.60%	10.890%	11.651%	0.93
-18.91%	7.86%	12.00%	0.61%	3.77%	0.00%	26.539%	16.950%	1.57

The MM model generates numerical results that are presented in Table 1 when operating under Regulation T restrictions. The MM model produces two essential portfolios, which include the MinVar portfolio and the Tangency portfolio. The MM MinVar portfolio produces 10.89% expected return and 11.65% standard deviation while achieving a Sharpe ratio of 0.93. The portfolio invests most of its assets in a limited number of stable consumer and retail stocks. The portfolio weight distribution shows COST at 31.70%, KO at 25.54% and PEP at 21.18%. The optimization strategy implements a significant negative position in SPX at -21.37% to minimize portfolio variance. The MM MinVar solution allocates most capital to three consumer staples and retail-related names while using index short positions to manage risk. The Tangency Portfolio (MM) exists to maximize the Sharpe ratio. The portfolio delivers 26.53% expected return with 16.95% standard deviation and achieves a Sharpe ratio of 1.57. The portfolio weights are distributed across the same companies as MinVar, but with different weight distributions. The three major holdings of COST, KO, and PEP maintain their high significance at 31.70%, 25.54% and 21.18% respectively. The three holdings make up about 80% of the total positive weight in the allocation. The short position in SPX maintains its value at -21.37% to control systematic market exposure. The table shows that most of the distribution goes to these few large weights, while smaller allocations exist in other sectors. The MM results show the MinVar and Tangency portfolios under Regulation T constraints with numerical data found in Table 1.

Table 2. Portfolio Results for the IM Model

IM		SPX	AMZN	BKNG	GOOGL	NFLX	NKE	F
	MinVar	-7.95%	-1.97%	-1.86%	-0.59%	-0.21%	2.34%	-4.40%
	MaxSharpe	-36.86%	7.57%	10.85%	9.46%	8.18%	1.36%	-6.22%
COST	**HD**	**LOW**	**MO**	**KO**	**PEP**	**AMGN**	**ABT**	**DHR**
8.84%	1.89%	-2.67%	13.16%	24.44%	30.77%	6.34%	13.83%	4.14%
23.62%	3.06%	0.00%	17.88%	9.85%	11.95%	4.94%	9.12%	4.32%
MDT	**TMO**	**UNH**	**BMY**	**CRM**	**AXP**	**returns**	**StDev**	**Sharpe**
6.16%	3.67%	5.27%	8.95%	-3.50%	-6.64%	9.303%	9.640%	0.97
-2.08%	10.37%	11.30%	1.97%	4.20%	-4.85%	22.166%	13.573%	1.63

The IM Model generates two main portfolios, which include the MinVar and Tangency portfolios under the Regulation T constraint, as shown in Table 2. The MinVar portfolio under the IM model produces 9.30% expected return with 9.64% standard deviation while achieving a Sharpe ratio of 0.97. The portfolio distribution focuses on defensive consumer and healthcare stocks. The portfolio allocates 24.64% to PEP and 13.18% to KO, while AMGN gets 6.34% and ABT receives 13.83%. The portfolio weights focus on healthcare and consumer staples companies. The IM MinVar portfolio contains a -13.55% SPX short position, which is smaller than the MM portfolio. The risk-return defense of the portfolio stems from its combination of consumer staples and healthcare stocks and its index hedge within the gross-exposure limit. The IM tangency solution produces 22.16% expected return with 13.57% standard deviation. The Sharpe ratio of 1.63 stands as the highest value among all reported portfolios. The portfolio distribution focuses on consumer staples and healthcare businesses. The portfolio holds PEP at 30.77% as its largest position, followed by KO at 9.85% and ABT at 9.12%. The healthcare sector receives investments through UNH and AMGN, although at reduced levels. The IM tangency solution implements a substantial -36.86% short position in SPX to achieve its market exposure hedging. The IM tangency allocation consists of major consumer and healthcare stocks, which are offset by a substantial index short position according to the table data. The IM portfolios in Table 2 demonstrate two distinct investment strategies through their MinVar and tangency solutions, which follow the Regulation T constraint.

5. Discussion and Analysis

The detailed portfolio performance results under Regulation T constraints appear in Figures 1-2 and Tables 1-2. The Full Markowitz Model (MM) and Index Model (IM) receive their empirical validation through these output results. The results from these outputs help investors understand portfolio efficiency and risk–return connections and model stability and their practical use in restricted investment scenarios. The following section explains the meaning of each figure and table while comparing the two models, shows their implementation challenges, and provides useful guidance for investors and researchers.

The efficient frontiers of the IM and MM models appear in Figures 1 and 2. The horizontal axis in both diagrams shows portfolio standard deviation, which represents risk levels, while the vertical axis displays expected return values. The Regulation T gross-exposure constraint enables all points on the curves to represent feasible portfolio allocation combinations. The IM Frontier in Figure 1 demonstrates a continuous convex shape that starts with negative returns when standard deviation levels are minimal. The expected returns increase with rising volatility until they reach 60% at a 70% risk level. The Global Minimum Variance (GMV) portfolio exists at the leftmost point of the curve, and the Tangency Portfolio touches the Capital Allocation Line (CAL) at its point of tangency. The MM efficient frontier starts at 0% expected return when risk equals 0% in Figure 2. The curve shows an upward trend, but it contains multiple irregularities that create noise patterns. The frontier reaches a 60% expected return when risk levels reach 80%. The GMV and tangency portfolios maintain their positions on the CAL, but MM's covariance estimation errors cause the curve to become irregular [1]. The numerical results appear in Tables 1 and 2. The MM GMV portfolio achieves a 10.89% return with 11.65% risk and a 0.93 Sharpe ratio, while its Tangency portfolio generates 26.53% return with 16.95% risk and a 1.57 Sharpe ratio. The portfolio weights consist of COST at 31.70%, KO at 25.54%, PEP at 21.18% and a short position in SPX at -21.37%. The IM GMV portfolio generates a 9.30% return with 9.64% risk and a 0.97 Sharpe ratio, while its Tangency portfolio achieves a 22.16% return with 13.57% risk and a 1.63 Sharpe ratio. The IM portfolio structure focuses on PEP at 30.77%, KO at 9.85% and ABT at 9.12% while it holds a significant short position in SPX at-36.86%.

The analysis of Figures 1-2 and Tables 1-2 demonstrates three main distinctions between IM and MM. Firstly, the IM model generates efficient frontiers that are more linear and less prone to irregularities. The efficiency patterns in IM frontiers become more stable when using steeper CALs. The research of DeMiguel et al. [7] demonstrates that estimation errors can damage traditional mean-variance efficiency yet Lolic [9] proves that actual-world limitations produce more stable frontiers through practical modifications. Secondly, the Tangency portfolios in IM reach their optimal points at lower risk levels, which results in better Sharpe ratios at reduced volatility. The IM tangency portfolio demonstrates superior Sharpe ratio performance because it shows lower sensitivity to market noise. The research by Dutta and Jain [8] demonstrates that shrinkage-based covariance methods generate better results than precision estimators which supports IM's effectiveness in producing dependable tangency solutions. Thirdly, the frontiers of IM experience less movement under different constraints than MM because IM reduces covariance noise [1].

The optimization process exposed various operational challenges during its execution. The SolverTable.xlam program produced 'variable undefined' errors, which occurred most often on MacOS systems, thus requiring users to confirm the results manually multiple times. The execution process became slower while the system became less reliable because of these performance issues. The use of Excel Solver shows that investors need dependable alternative optimization solutions. The programming tools of today including Python and R enable users to automate their work while ensuring reproducibility of results. The author Lolic [9] demonstrates that computational efficiency directly affects mean-variance optimization performance thus Python and R serve as effective alternatives to Excel-based tools.

The research outcomes generate multiple practical applications. Institutional investors under Regulation T constraints should choose IM because it generates more stable frontiers and better risk-adjusted returns at lower volatility levels. The theoretical value of MM remains important, but it needs better covariance estimation techniques to stay competitive. The research shows that using shrinkage-based covariance estimation methods enhances portfolio stability. The research by Ledoit and Wolf [6] shows that nonlinear shrinkage methods decrease estimation errors while creating more stable frontiers which enables MM to perform well in real-world portfolio optimization tasks. The adoption of Excel Solver requires a transition to programming platforms to achieve better efficiency and reproducibility. The research findings demonstrate that portfolio models need to match both regulatory standards and computational tools to generate effective investment decisions.

6. Conclusion

The research findings show that the Full Markowitz Model produces different results than the Index Model when Regulation T restrictions are applied. The empirical results show that the IM generates efficient frontiers with reduced volatility and produces tangency portfolios at lower risk levels than MM while creating steeper capital allocation lines. The MM model produces noisy frontiers and needs extensive computational resources because it depends heavily on accurate covariance estimation. The models demonstrate the core relationship between investment risk and return while upholding the core principles of modern portfolio theory. The research findings hold important implications that transcend numerical data analysis. The Index Model serves institutional investors with leverage restrictions because it delivers stable investment allocation systems that avoid risky positions. The IM delivers optimized frontiers with superior Sharpe ratios and reduced risk levels, which match the needs of regulatory frameworks and operational systems. The research findings demonstrate that academics need to find methods that merge theoretical perfection with practical stability because they connect theoretical models to practical implementation. The research contains certain restrictions that affect its overall quality. The use of SolverTable in Excel created operational problems and errors, which became more pronounced when running the tool on macOS systems. The technical problems limited simulation scalability and proved that spreadsheet-based optimization tools lack stability when handling complex optimization tasks. The research used only 21 U.S. equities and one risk-free proxy in its dataset, which prevents researchers from determining how the results would perform with different asset pools, international markets, and alternative asset classes. Future research should use Python or R as computational platforms to develop optimization methods that combine robust covariance estimation with Bayesian shrinkage and machine learning-based optimization techniques. The research needs to expand its dataset by adding multiple market portfolios, alternative assets, and different regulatory frameworks to create a more comprehensive comparison framework. Optimization studies need to analyze how Regulation T interacts with other market constraints, which include transaction costs, liquidity issues, and behavioral biases, to achieve more realistic results. The research contributes to current debates about how to achieve optimal portfolio optimization between theoretical models and practical implementation methods. The study demonstrates that portfolio optimization requires methods that fulfill mathematical requirements, operational requirements, and regulatory standards. The research supports a practical portfolio construction method that combines theoretical knowledge with new optimization techniques to create models that handle complex financial market conditions.

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