Risk Budget Management under a Factor Investment Framework: Theoretical Extension and Application in Target Volatility Strategies

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Abstract. In recent years, China's stock market has transitioned between bear and bull markets. Smart Beta strategies combine the strengths of active and passive investing, effectively enhancing risk control and return generation capabilities in asset allocation. Although this strategy has achieved significant scale in foreign financial markets, fund products based on Smart Beta strategies remain relatively small in scale domestically. This study utilizes approximately five years of daily data (from 2021 to present) from representative indices of various style factors. Employing the sequential least squares algorithm, it constructs a four-factor portfolio combining low-correlation factors with strong risk-hedging capabilities and the green finance ESG factor—suitable for long-term development. This approach validates the limited scope of integrating ESG factors into asset allocation while determining the optimal weightings for this four-factor portfolio across different expected volatility ranges. The findings demonstrate that incorporating ESG factors can indeed optimize asset allocation outcomes. However, portfolio weights must be dynamically adjusted according to target volatility levels to achieve effective risk control capabilities during implementation.

Keywords: Smart Beta, ESG, Asset Allocation, Risk Management.

1. Introduction

With the increasing sophistication of asset allocation practices and the rapid growth of index investing, factor investing has emerged as a vital tool in modern portfolio management. Smart Beta strategies aim to achieve risk-adjusted returns that outperform traditional market-cap-weighted indices by systematically exposing investors to specific risk factors. By the end of 2024, the U.S. market had amassed approximately $2.5 trillion in Smart Beta ETF assets under management, with 650 products available. In contrast, China's market size stood at only about RMB 140.781 billion, featuring 75 products. Notably, China's policy framework explicitly supports the development of strategy-based index products like dividend and low-volatility indices, creating a favorable environment for advancing Smart Beta ETFs.

Despite the growing body of research on smart beta strategies, most existing literature remains focused on single-factor validity testing or static multi-factor portfolio construction. In fact, creating multiple weighted versions of the CSI 300 Index ETF not only diversifies investment options for investors with different risk preferences but also helps establish reasonable expectations for capital market risks, thereby promoting the healthy development of the capital market [1].

However, integrating factor investing with dynamic risk objectives and designing customized allocation solutions for investors with varying risk preferences remains a research gap. Particularly in domestic markets, where product homogeneity (e.g., concentration in dividend strategies) coexists with insufficient strategy complexity, exploring optimal multi-factor allocations under target volatility constraints holds significant theoretical innovation and practical value.

Considering the cyclical fluctuations in the Chinese market, as exemplified by the CSI 800 index, the overall decline during the pandemic period from 2020 to 2023 was 11.96%, with a volatility of 1.18%, an average daily fluctuation of 1.40%, and a maximum drawdown reaching 64.61% (highest point: 6,016.50 points, lowest point: 3,620.36 points). During the minor bull market in mid-2025 (June), the CSI 800 rose by 2.96%, with volatility significantly declining to 0.62% and average daily fluctuation reaching 0.84%. Market turnover hit 8.82 trillion yuan, indicating heightened market activity.

To mitigate high volatility stemming from systemic and non-systemic risks, this study prioritizes low-correlation factors for allocation to effectively hedge risks. Different return models yield tangency portfolio weights with distinct characteristics—some highly diversified (such as equal-weighted portfolios), while others are highly concentrated [2]. Therefore, how to screen configuration objects and how to adjust weights becomes a serious issue.

Since the integration approach may narrow the investable universe to a set of stocks that cannot construct a highly diversified portfolio [3], this study employs factor indices as proxies for allocation to achieve a hybrid strategy effect.

Moreover, corporate green transformation can significantly enhance the financial efficiency of asset allocation. It can also alleviate financing constraints by optimizing internal asset allocation and reducing corporate credit risk, thereby improving the efficiency of financial asset allocation. Additionally, enterprises can leverage green transformation to reduce agency costs arising from information asymmetry between shareholders and management, refine investment structures, and boost the efficiency of financial asset allocation [4].

Consequently, this study deliberately incorporates ESG investment principles. As an emerging strategy increasingly favored by markets and investors, ESG is gradually transforming traditional investment models. Its core lies in deeply integrating environmental, social, and corporate governance—three non-financial factors—into portfolio selection and management processes, thereby constructing a more balanced and sustainable investment portfolio [5].

Meanwhile, ESG factors demonstrate pricing efficiency in the A-share market. Compared to performance benchmarks, long-only portfolios constructed from stocks with the highest ESG composite scores generate significant excess returns. These portfolios also perform well during bear markets, exhibiting strong risk control capabilities [6]. Furthermore, while ESG factors exhibit relatively high first-order autocorrelation coefficients, their correlations with other traditional major factors remain below 0.40. They even show negative correlations with size, momentum, and growth factors, a characteristic that facilitates the construction of low-correlation portfolios [7]. ESG factors also align with green finance principles, making them more likely to receive policy support.

Empirical research indicates that most factor returns do not stem from traditional market risk exposures. Their long-term performance exhibits a significant negative market beta characteristic, with the majority of primary returns originating from bear markets [8]. This finding reveals that factor returns possess unique risk-return characteristics and underscores the necessity for effective risk control over factor portfolios in asset allocation.

This study aims to combine currently effective low-correlation factors with investment-viable ESG factors to construct a four-factor model for portfolio allocation. By integrating ESG factors with three low-volatility factors and employing a constrained optimization model, the objective function targets weight variance minimization (pursuing balanced allocation). Constraints include precisely matching portfolio volatility to an exogenous target and weight restrictions (e.g., ESG $\leq$ 50%). to determine optimal factor weights across different risk levels (target volatility constraints). This approach enhances risk control during bear markets while mitigating potential utility decay in bull market environments.

In terms of research methodology, we first validate the model's feasibility under varying target volatilities. Subsequently, based on computational results and historical data, we identify the upper and lower bounds for factor allocations under specific volatility targets tailored to conservative, balanced, and aggressive investors. This ultimately provides precise allocation guidance for differentiated risk preferences.

2. Method

2.1. Low-Correlation Factor Screening

2.1.1 An Intuitive Comparison of Sharpe Ratio And Correlation

Using an equal-weighted calculation method, we computed the annualized average return, volatility, and Sharpe ratio for each pair of assets (28 pairs in total). We then visualized the Sharpe ratio alongside correlation to make preliminary assessments. In the visualized scatter plot, pairs exhibiting higher Sharpe ratios coupled with lower correlations align more closely with the criteria of high returns and low volatility.

2.1.2 Ranking Scoring System Comparison

Since the visualization chart uses the Sharpe ratio instead of yield and volatility, it cannot directly display specific yield and volatility values and should only be used as a preliminary reference.

Therefore, this study also employs a ranking-based scoring system. The annualized average return, volatility, and correlation coefficient of each index are ranked separately (returns are negatively correlated with ranking values, while volatility and correlation coefficients are positively correlated with ranking values). Scores are assigned using the formula (where represents rank and N represents total number). When calculating the total score, considering the dual nature of volatility and the higher priority of low correlation coefficients, this study applies the weighting "correlation coefficient: return: volatility = 4.5:3.5:2" for statistical analysis, retaining only portfolios with a total score exceeding 0.5.

2.2. ESG Factor Data Acquisition

Given the representativeness issues of single-sector ESG indices and the complexity of obtaining comprehensive historical ESG rating data, this study constructs a composite ESG index for analysis by screening ESG stock indices covering multiple sectors and calculating their average values.

2.3. Asset Allocation

2.3.1 Equal Weighting Method

Equal-weighting assigns equal importance to each stock (or index), allocating an equal 1/N weight to each of N stocks (or indices) in the portfolio. This method provides a quick overview of the returns and volatility levels across different asset combinations.

However, traditional market-cap-weighted indices assign greater weight to large-cap stocks, whereas equal-weight index strategies significantly increase the weight of small- and mid-cap stocks. Historical data indicates that in most markets, small-cap portfolios often outperform large-cap portfolios [9]. This may lead to the risk differences between assets being overlooked, resulting in unstable overall portfolio risk levels.

2.3.2 Risk-Parity Weighting Method

Risk parity is a risk allocation strategy that balances the risk contributions of various assets to the overall portfolio. For a single asset allocation, the weighting vector of individual stocks in the portfolio is w, and the covariance matrix is Ω. The allocation criteria are as follows:

$$\min f(w)=\sum_{i=1}^{N}\sum_{j=1}^{N}\left[w_i(\Omega w_i)-w_j(\Omega w_j)\right]^2 \tag{1}$$

$$\text{s.t.}\begin{cases} w'1=1 \\ w_i\geq 0 \end{cases} \tag{2}$$

However, when calculated solely from the perspective of optimizing volatility, it may lead to overly conservative allocations when asset volatilities vary significantly.

2.3.3 Combined Optimization Of Methods

This study employs the Sequential Least Squares Programming (SLSQP) optimization algorithm, a sequential quadratic programming method specifically designed for solving nonlinear programming problems. It generates search directions by solving linear constrained least-squares subproblems and enhances computational efficiency through direct matrix decomposition rather than approximate Hessian matrix methods. This approach efficiently locates optimal weight combinations that satisfy all constraints while minimizing the objective function, making it suitable for solving similarly constrained nonlinear optimization problems [10]. This study establishes a dual-objective optimization framework, employing rigorous mathematical programming to achieve the predetermined target volatility.

First is volatility characterization. The volatility of an asset allocation is jointly determined by the asset weight vector w and the covariance matrix Ω.$\boldsymbol{\sigma_p}$ represents the actual portfolio volatility under different target volatilities. The calculation formula is:

$$\sigma_p=\sqrt{w'\Omega w} \quad (3)$$

Next is the objective function optimization. The portion before the plus sign in this formula ensures that the actual volatility value of the portfolio combination aligns with the target volatility, preventing sharp jumps in allocation results across different target volatilities and making the output more suitable for actual holdings. Multiplying by 1000 specifies that achieving the target volatility takes priority over minimizing weighted variance. The standard allocation formula is:

$$\min J(w)=1000\times(\sigma_p-\sigma_{target})^2+Var(w) \quad (4)$$

Finally, the constraint settings are as follows: Constraint 1 requires the total weight sum to equal 1. Constraint 2 sets the upper and lower bounds for the weights of the three low-correlation factors within the range $\mathbf{0\leq w_i\leq 1}$. Considering that the ESG factor is in the experimental configuration phase, its weight bounds are also set to $\mathbf{0\leq w_{ESG}\leq 0.5}$.

The algorithm incorporates a fallback mechanism to ensure computational stability: if optimization fails, it defaults to an equal-weight configuration; if convergence still cannot be achieved, it allocates 100% to low-wave indices. Fig 1 depicts the algorithmic flowchart for the optimization model:

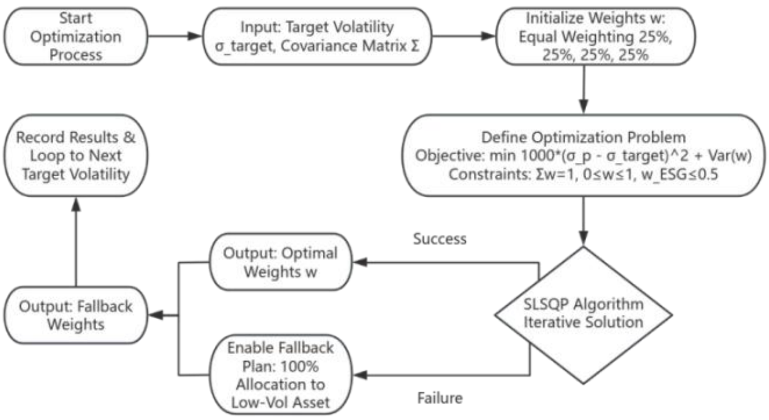

Figure 1. The algorithmic flowchart for the optimization model

2.4. Efficiency Comparison of Factors Associated With The Three Lows

The performance evaluations employed in this study are based on historical data backtesting and assume that the expected annual returns of the portfolio follow a normal distribution.

2.4.1 Calculation Standard

Confidence Intervals for Returns (50% and 95% Probability): Under the assumption that future returns follow a normal distribution, these intervals indicate the range within which future returns are likely to fall.

Probability of achieving positive returns: This provides an intuitive representation of the likelihood, based on historical experience, that investing in this portfolio will yield positive returns after one year.

Average Annualized Return: The portfolio's average annual return during the backtesting period.

2.4.2 Comparative Standard

Based on the assumption of normal distribution, this study employs $X_{Four\ factors}$-$X_{Three\ factors}$ to calculate the upper and lower bounds of the yield, and $\frac{(X_{Four\ factors}-X_{Three\ factors})}{|X_{Three\ factors}|}$ to calculate the average yield and historical positive yield.

2.5. Calculate The Upper and Lower Limits For The Allocation of The Three Target Volatilities Selected Through Screening

To determine the upper and lower bounds for factor allocations across three target volatility levels, this study first calculates the maximum, minimum, and range of allocation weights over the entire data period. Subsequently, using the quartile points as boundaries, the allocation range was divided into three tiers: ‘Underweight’ (from the lower limit to the 1st quartile), ‘Standard Allocation’ (from the 1st to the 2nd quartile), and ‘Overweight’ (from the 2nd quartile to the upper limit).

3. Calculation Process And Results Presentation

The time span of data selected for this study is from January 4, 2021, to September 2, 2025. Data is sourced from Wind, primarily because many ESG fund indices were released relatively late. With a small sample size, the reference value of the resulting indices would be low. Additionally, this period encompasses both the bear market driven by China's economic downturn due to COVID-19 and the minor bull market in 2025 marked by accelerated overall economic growth. This effectively covers data from different periods, ensuring higher reliability.

3.1. Screening Of Factors Associated With The Triple Low

The factor pool for this study includes: Size factor-CSI 800 index, Low-Vol factor-CJSC Low Volatility Index, Dividend Yield factor-CJSC Dividend Index, MOM factor-CJSC Momentum Index, Growth factor-CJSC Growth Index, Quality factor-CJSC High Earnings Quality Index, Small-Cap factor-CJSC Small-Cap Index, Large-Cap factor-CJSC Large-Cap Index (Factor Name - Corresponding Index Representation - Wind Code).

First, obtain the daily percentage changes for all indices between January 4, 2021, and September 2, 2025. Subsequently, calculate the correlation coefficient matrix among the factors, presented in Table 1. Concurrently, calculate their annualized average returns, volatility, and Sharpe ratios, presented in Table 2:

Table 1. The correlation coefficient matrix

correlation	Size	Low-Vol	Dividend Yield	MOM	Growth	Quality	Small-Cap	Large-Cap
Size								
Low-Vol	0.7209							
Dividend Yield	0.7578	0.8414						
MOM	0.7170	0.3803	0.7871					
Growth	0.8023	0.3863	0.5390	0.8414				
Quality	0.9173	0.5645	0.5979	0.6310	0.7039			
Small-Cap	0.8220	0.6562	0.8044	0.7718	0.8134	0.6334		
Large-Cap	0.9876	0.6852	0.6964	0.6785	0.7681	0.9361	0.7388	

Table 2. Annualized average return volatility and Sharpe ratio

	Size	Low-Vol	Dividend Yield	MOM	Growth	Quality	Small-Cap	Large-Cap
annualized average returns	36.448%	18.028%	22.556%	60.805%	82.055%	28.476%	56.841%	33.076%
stdev	22.555%	17.714%	20.467%	34.323%	35.256%	24.121%	30.757%	21.560%
sharpe	1.6160	1.0177	1.1021	1.7716	2.3274	1.1805	1.8481	1.5341

An intuitive comparison of Sharpe and Correlation ratios, as shown in Fig. 2:

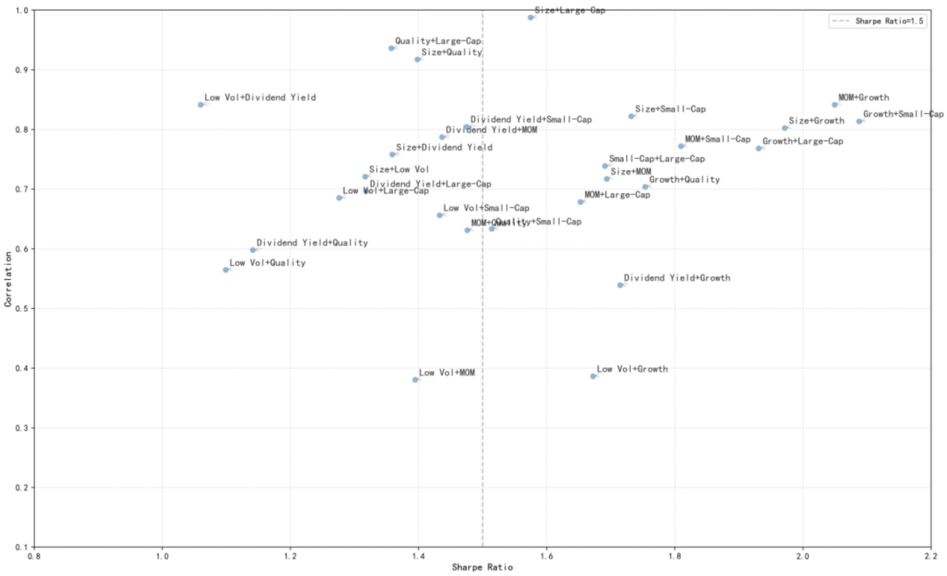

Figure 2. Intuitive comparison of Sharpe and Correlation ratios

Based on the derived characteristics inferred from the original standards—namely high Sharpe ratio and low correlation—it is preliminarily concluded that the three-factor combinations of low volatility with growth, low volatility with momentum, and dividends with growth are relatively well-suited to the requirements.

Subsequently, the results obtained through the ranking-based scoring system in this study are presented in Fig. 3:

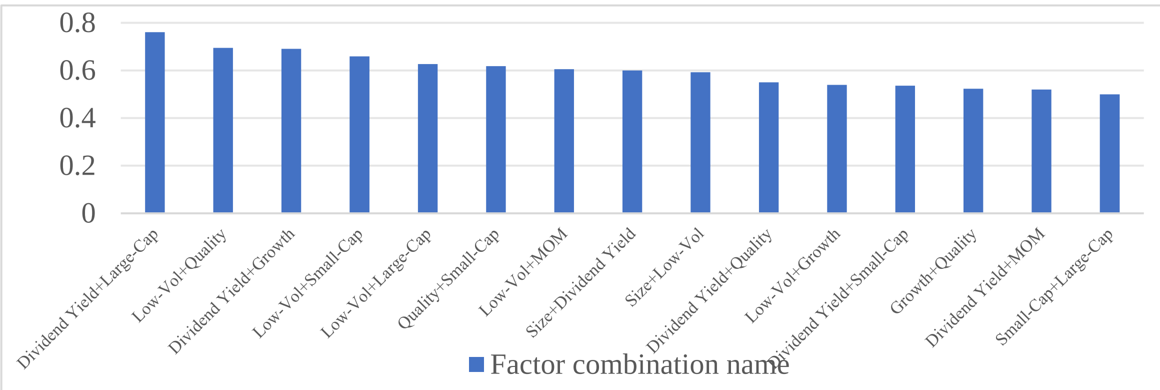

Figure 3. Ranking and scoring system calculation results

Finally, the two-factor combinations identified through the above screening process were used to calculate three-factor combination scores. Specifically, the average values of the corresponding three pairs of two-factor combinations were computed and subsequently ranked. Table 3 presents the top three rankings based on the calculated total scores:

Table 3. The top three in the overall score ranking

Low-Vol+MOM+Quality	0.686904762
Quality+Small-Cap+MOM	0.613690476
Dividend Yield+Large-Cap+MOM	0.610714286

Additionally, this combination includes the dual-factor combination Low-Vol+MOM identified in the preliminary assessment above. Therefore, the three low-correlation factors selected for this analysis are Low-Vol+MOM+Quality.

3.2. Calculation Of ESG Factors

This study calculated the ESG indices used in the experiment by averaging 34 Chinese ESG stock indices covering multiple industries.

3.3. Four-Factor Configuration

Subsequently, the volatility of the three low-correlation factor representative indices relative to the ESG provisional index was calculated, yielding an approximate volatility range of 14.1%–28.2% for this portfolio, as shown in Fig. 4:

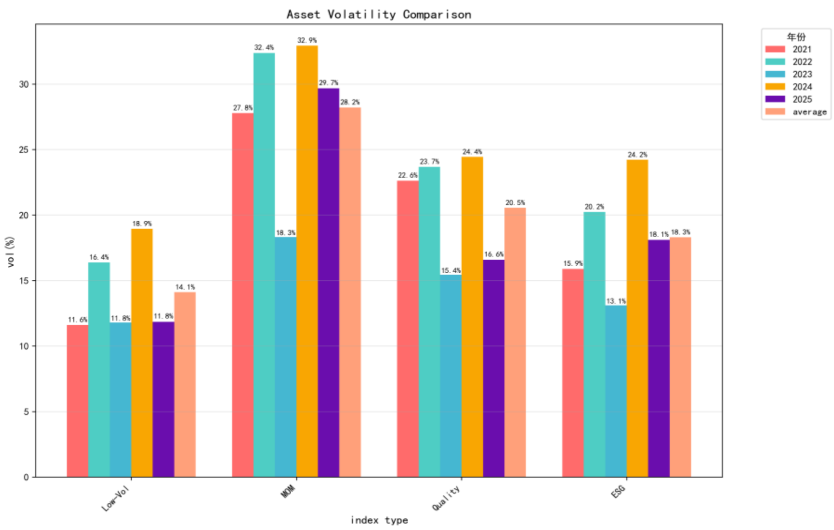

Figure 4. Asset Volatility Comparison

The configuration of asset combinations may cause fluctuations in the effective range of the final portfolio volatility. Therefore, when planning the target volatility, a calculation interval of [13%, 29%] with a step size of 1% was selected. Sequential least squares programming (SLSQP) optimization was performed, yielding the asset allocation results shown in Table 4:

Table 4. Asset Allocation Results

ESG	Low-Vol	MON	Quality	Realized Vol	Target Vol	allocation stdev
25.00%	25.00%	25.00%	25.00%	17.39%	13%	0.00%
16.63%	57.75%	0.18%	25.45%	14.00%	14%	24.21%
21.83%	44.68%	8.28%	25.21%	15.00%	15%	15.02%
23.81%	35.31%	15.60%	25.27%	16.00%	16%	8.09%
24.75%	27.70%	22.42%	25.13%	17.00%	17%	2.16%
25.31%	21.09%	28.85%	24.75%	18.00%	18%	3.18%
25.56%	15.21%	35.07%	24.17%	19.00%	19%	8.13%
25.67%	9.82%	41.07%	23.43%	20.00%	20%	12.80%
25.67%	4.83%	46.94%	22.56%	21.00%	21%	17.27%
25.63%	0.14%	52.68%	21.54%	22.00%	22%	21.58%
22.92%	0.00%	61.25%	15.82%	23.00%	23%	26.00%
20.54%	0.00%	69.10%	10.36%	24.00%	24%	30.57%
18.45%	0.00%	76.37%	5.18%	25.00%	25%	35.12%
16.54%	0.00%	83.23%	0.23%	26.00%	26%	39.58%
9.09%	0.00%	90.91%	0.00%	27.00%	27%	44.15%
1.56%	0.00%	98.44%	0.00%	28.00%	28%	48.97%
0.00%	0.00%	100.00%	0.00%	28.21%	29%	50.00%

The numerical trends of factor allocation weights and weight variances appear largely reasonable, but outliers requiring exclusion have emerged: First, when the target volatility is 13%, the optimization process resulted in equal weighting due to its inability to achieve the expected target volatility; Second, when the target volatility is 29%, the actual volatility only reaches 28.21%. This is due to the maximum volatility constraint on individual holdings within the portfolio, where the momentum index allocation reaches 100%, deviating from conventional asset allocation practices. Therefore, the final effective target volatility range is [14%, 28%].

3.4. Efficiency Comparison Between Four-Factor and Three-Low-Related Factor Allocations

This study selected three low-correlation factors as the control group. Since the approximate volatility range for this combination is also 14.1%–28.2%, a more convenient target volatility range [14%, 28%] can be used for SLSQP optimization calculations. After determining the corresponding allocation ratios, the allocation effects of both the four-factor and three-low-correlation-factor strategies were separately evaluated. The results of the four-factor configuration and the comparison of computational efficiency for the normal distribution are shown in Tables 5 and 6, respectively:

Table 5. Calculation results of four factors

growth rate-50% probability - upper limit of return	growth rate-50% probability - lower limit of return	growth rate-95% probability - upper limit of return	growth rate-95% probability - lower limit of return	growth rate-historical positive probability	growth rate-average return	growth rate-target volatility
12.42%	-6.34%	30.48%	-24.40%	58.58%	3.04%	14%
13.26%	-6.84%	32.61%	-26.19%	58.48%	3.21%	15%
14.12%	-7.32%	34.76%	-27.96%	58.41%	3.40%	16%
15.01%	-7.77%	36.94%	-29.70%	58.44%	3.62%	17%
15.94%	-8.18%	39.16%	-31.40%	58.53%	3.88%	18%
16.89%	-8.57%	41.40%	-33.08%	58.66%	4.16%	19%
17.86%	-8.94%	43.66%	-34.74%	58.82%	4.46%	20%
18.84%	-9.30%	45.93%	-36.39%	58.99%	4.77%	21%
19.84%	-9.64%	48.22%	-38.02%	59.17%	5.10%	22%
21.63%	-9.19%	51.30%	-38.86%	60.67%	6.22%	23%
23.36%	-8.80%	54.32%	-39.76%	61.92%	7.28%	24%
25.02%	-8.48%	57.27%	-40.73%	62.96%	8.27%	25%
26.64%	-8.20%	60.18%	-41.74%	63.85%	9.22%	26%
27.78%	-8.40%	62.61%	-43.23%	64.01%	9.69%	27%
28.88%	-8.64%	65.00%	-44.76%	64.12%	10.12%	28%

Table 6. The comparison of computational efficiency for the normal distribution

Difference rate-50% probability - upper limit of return	Difference rate-50% probability - lower limit of return	Difference rate-95% probability - upper limit of return	Difference rate-95% probability - lower limit of return	Difference rate-historical positive probability	Difference rate-average return	rate-target volatility
0.41%	0.41%	0.41%	0.41%	1.12%	0.41%	14%
0.36%	0.36%	0.36%	0.36%	0.94%	0.36%	15%
0.28%	0.28%	0.28%	0.28%	0.67%	0.28%	16%
0.19%	0.19%	0.19%	0.19%	0.45%	0.19%	17%
0.13%	0.13%	0.13%	0.13%	0.28%	0.13%	18%
0.08%	0.08%	0.08%	0.08%	0.16%	0.08%	19%
0.03%	0.03%	0.03%	0.03%	0.06%	0.03%	20%
-0.01%	-0.01%	-0.01%	-0.01%	-0.02%	-0.01%	21%
-0.05%	-0.05%	-0.05%	-0.05%	-0.08%	-0.05%	22%
0.71%	0.71%	0.71%	0.71%	1.20%	0.71%	23%
1.39%	1.39%	1.39%	1.39%	2.23%	1.39%	24%
1.48%	1.48%	1.48%	1.48%	2.26%	1.48%	25%
1.30%	1.30%	1.30%	1.30%	1.88%	1.30%	26%
0.70%	0.70%	0.70%	0.70%	0.97%	0.70%	27%
0.11%	0.11%	0.11%	0.11%	0.16%	0.11%	28%

According to Table 6, it can be concluded that except for target volatilities of 22% and 23%, the four-factor strategy outperforms the three low-correlation factors. Even at target volatilities of 24%, 25%, and 26%, the indicators demonstrate strong profitability. For instance, at a 25% target volatility, the average return indicator increased by 1.48%. while also demonstrating outstanding volatility control performance (95% probability upper/lower bounds of returns increased by 1.30%).

3.5. Screening Representative Target Volatility

The results of the index differences between target volatilities (e.g., values at 15% volatility minus values at 14% volatility) are presented in Table 7:

Table 7. Results of the index spread between target volatilities

growth rate-50% probability - upper limit of return	growth rate-50% probability - lower limit of return	growth rate-95% probability - upper limit of return	growth rate-95% probability - lower limit of return	growth rate-historical positive probability	growth rate-average return	rate-target volatility
-	-	-	-	-	-	14%
0.84%	-0.50%	2.13%	-1.79%	-0.10%	0.17%	15%
0.86%	-0.48%	2.15%	-1.77%	-0.07%	0.19%	16%
0.89%	-0.45%	2.18%	-1.74%	0.03%	0.22%	17%
0.93%	-0.41%	2.22%	-1.70%	0.09%	0.26%	18%
0.95%	-0.39%	2.24%	-1.68%	0.13%	0.28%	19%
0.97%	-0.37%	2.26%	-1.66%	0.16%	0.30%	20%
0.98%	-0.36%	2.27%	-1.65%	0.17%	0.31%	21%
1.00%	-0.34%	2.29%	-1.63%	0.18%	0.33%	22%
1.79%	0.45%	3.08%	-0.84%	1.50%	1.12%	23%
1.73%	0.39%	3.02%	-0.90%	1.25%	1.06%	24%
1.66%	0.32%	2.95%	-0.97%	1.04%	0.99%	25%
1.62%	0.28%	2.91%	-1.01%	0.89%	0.95%	26%
1.14%	-0.20%	2.43%	-1.49%	0.16%	0.47%	27%

Decision Logic: Reject absolute value comparisons and focus on "marginal improvement"—the incremental return gain per unit of increased risk (volatility).

3.5.1 Conservative

Choice: 16%.

Reason: The most significant improvement in robustness comes from the unit of risk.

Within the low-risk range of 14%-20%, a target volatility of 16% offers the optimal risk-return tradeoff efficiency. Its "average return growth rate" is 0.17%, meaning that compared to 15% volatility, each additional 1% of risk increases the expected return by 0.17%. Simultaneously, its "95% probability lower bound growth rate" (worst-case improvement) stands at -1.77%, outperforming the adjacent 15% (-1.79%) and 17% (-1.74%) levels. This indicates that assuming additional risk at this point yields more effective downside risk control and enhanced returns.

3.5.2 Balanced

Choice: 20%.

Reason: The peak of risk-return tradeoff.

20% volatility threshold marks the endpoint of incremental improvements within the low-to-medium risk range. Beyond this point, growth rates across metrics begin to level off (e.g., the average return growth rate at 21% is only 0.31%). Selecting 20% means this target volatility point captures all systemic improvement dividends within the 14%-20% range while avoiding the subsequent phase of diminishing marginal returns. It provides investors with a solid performance foundation, representing the perfect equilibrium between risk and return enhancement.

3.5.3 Ambitious

Choice: 23%.

Reason: The inflection point of marginal improvement, offering the highest value for money.

At 23% volatility, the "average return growth rate" surges to 1.12%, while its "historical positive return probability growth rate" also reaches 1.12%—far surpassing all other options. This signifies that at this point, each 1% increase in volatility yields over 1% in average returns and positive probability improvement, achieving peak efficiency in the risk-return tradeoff. Although 24% offers higher absolute returns, its marginal improvement (1.06%) begins to decline. Thus, 23% represents the optimal position to capture the strongest improvement momentum, delivering the highest value for money for aggressive investors.

3.6. Calculation of High-Standard Low-Allocation

The maximum and minimum allocation ratios and allocation ranges for conservative, balanced, and aggressive strategies are shown in Fig 5:

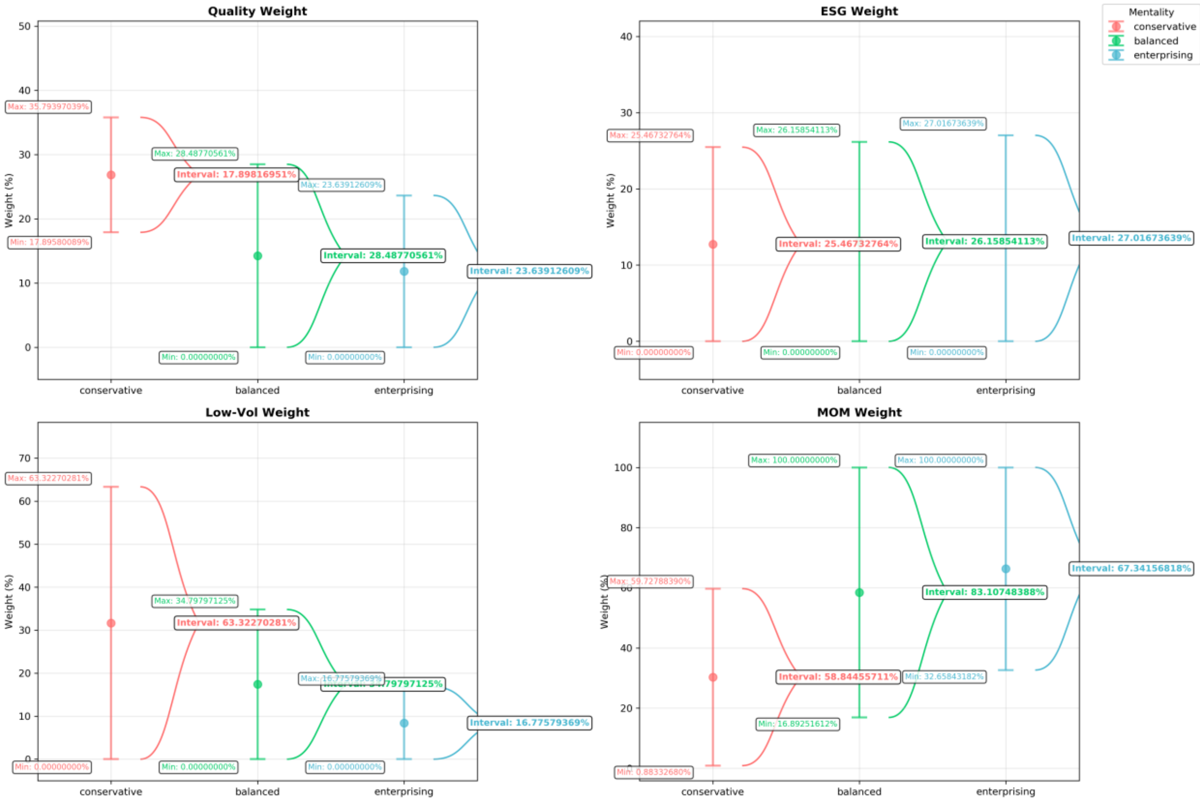

Figure 5. Weight Configuration Analysis by Mentality

The results of the high-standard, low-specification calculation are shown in Table 8:

Table 8. Calculation Results for High, Standard, and Low Configurations

Target Volatility	configuration	Low_Vol Lower limit	Low_Vol Upper limit	MOM Lower limit	MOM Upper limit	Quality Lower limit	Quality Upper limit	ESG Lower limit	ESG Upper limit
	low	0.000	21.108	0.883	20.498	17.896	23.862	0.000	8.489
target Vol 16%	standard	21.108	42.215	20.498	40.113	23.862	29.828	0.000	16.978
	high	42.215	63.323	40.113	59.728	29.828	35.794	16.978	25.467
	low	0.000	11.599	16.893	44.595	0.000	9.496	0.000	8.720
target Vol 20%	standard	11.599	23.199	44.595	72.298	9.496	18.992	0.000	17.439
	high	23.199	34.798	72.298	100.000	18.992	28.488	17.439	26.159
	low	0.000	5.592	32.658	55.106	0.000	7.880	0.000	9.006
target Vol 23%	standard	5.592	11.184	55.106	77.553	7.880	15.759	0.000	18.011
	high	11.184	16.776	77.553	100.000	15.759	23.639	18.011	27.017

4. Discussion

According to the table analysis, incorporating ESG into the allocation of the three low-related factors can indeed effectively enhance the profitability of the factor portfolio (average return, upper bound of returns, and lower bound of returns). Combined with its core green philosophy enabling long-term stable development, it is evident that ESG factors hold a certain degree of effective asset allocation status in the Chinese market.

However, by examining Tables 5 and 7, this study observes that the probability of positive returns also increases significantly with rising volatility. Even at a target volatility of 23%, the growth rate of the positive return probability reaches 1.50%. Why is this the case?

By combining Table 2 and Table 4, this study reveals that the "Momentum" factor (potentially combined with the "ESG" factor) not only exhibits high volatility (34.323%) but also boasts an exceptionally high historical average return (60.805%). Consequently, when the SLSQP algorithm is compelled to substantially increase allocations to the high-risk, high-return momentum factor to meet elevated volatility targets, the portfolio's expected return is correspondingly boosted. This leads to an increase in the probability of positive returns. For instance, when the target volatility rises from 22% to 23%, the allocation growth rate for Momentum Over Momentum (MOM) increases by 8.57%, while the growth rate of positive returns peaks at 1.50%.

Additionally, the 22%-23% occurrence of brief negative "anomalies" in Table 6 corroborates this logic (i.e., the four-factor model underperforms the three-factor model).

Additionally, the 22%-23% "abnormal" values corroborate this logic, which may even explain why temporary negative improvement values (i.e., four-factor underperformance relative to three-factor) occur within the 22%-23% volatility range.

This may be because the algorithm is undergoing a "transition period" for weight switching within this specific narrow volatility range. While experimenting with different weight combinations to precisely match higher volatility, the weight allocation failed to fully leverage the momentum factor's high-risk, high-return advantage, even resulting in a slight negative effect. However, after breaking out of this range (24%), the algorithm discovered a stable allocation scheme that maximizes the momentum factor's strengths, causing the difference to revert to positive territory.

The following logic can be derived:

Higher target volatility→The algorithm is forced to overweight high-volatility factors (momentum)→This factor inherently possesses a high positive return expectation→The probability of positive returns for the entire portfolio consequently increases.

This algorithm systematically captures specific risk premiums (such as the momentum premium) through strict rules and constraints.

5. Conclusion

First, the risk estimation dimensions of the model can be further expanded. This study primarily relies on target volatility as the core risk constraint, but in practice, additional metrics such as downside risk (e.g., Conditional Value at Risk, maximum Drawdown, factor crowding) should also be considered. For instance, the outliers observed in the 22%-23% volatility range may indicate limitations of a pure volatility constraint. Introducing multidimensional risk constraints thus helps identify portfolio vulnerabilities under extreme market conditions—such as a sudden surge in factor correlations—enhancing the robustness of optimization outcomes.

Second, the consideration of factor timing and dynamic weighting is somewhat inadequate. The current optimization does not incorporate macroeconomic variables or market state variables (such as interest rate cycles, credit spreads, or momentum factor crowding). For instance, the momentum factor may exhibit structural changes in high-volatility environments, and its Momentum Crash during bear markets may not be fully captured. Therefore, market state recognition could be integrated into the optimization framework, allowing factor weights to dynamically adjust based on the macroeconomic environment. This would help avoid overexposure to certain factors during adverse market conditions.

Furthermore, the ESG indices employed in this study are provisional indices derived from screening and averaging components. Given that many ESG indices have been in operation for a relatively short duration, the data only covers approximately the past five years, limiting its reference value. Concurrently, the indices could be optimized by incorporating time series models, such as autoregressive models, thereby enhancing the precision and effectiveness of the calculations.

Finally, the integrity of the backtesting framework can be further enhanced. Current results focus on static optimization and statistical metric comparisons, lacking out-of-sample backtesting and analysis of turnover rate/transaction cost impacts. It is recommended to construct a dynamically allocated portfolio using a rolling window approach, evaluate its performance on historical out-of-sample data, and account for the erosion of final returns caused by transaction costs—particularly the potentially high turnover rates associated with momentum factors. This will ensure that research conclusions possess not only statistical significance but also practical applicability.

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