Crop planting strategies under robust linear programming based on constrained approximation

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Abstract. In the context of rural revitalization and agricultural modernization, the optimization of crop cultivation strategies is crucial to enhancing farmers' incomes and promoting sustainable economic development. Although the existing strategies are continuously optimized, the consideration of crop, land management and climate factors is still insufficient. Therefore, this paper proposes a crop planting strategy under robust linear programming based on constrained approximation, aiming at combining rural reality, scientifically allocating crop and plot resources, and formulating an optimal planting plan to maximize the planting income in the next seven years. Firstly, different crops and their planting plot types are summarized and processed, and the constraints are constraint-approximated and equated, and secondly, according to the distribution of crops, a variety of plot types are subdivided and the constraints are reduced accordingly.Finally, set the decision variable for the plant's planting area, for a certain quarter of a plant's planting per acre profit, for different chunks, consider and select the appropriate constraints, introduce a linear programming model, bring in the objective function, for more than part of the stagnant sales, resulting in a waste of the situation through the MATLAB to establish a linear programming model to maximize the total return, develop the optimal planting program and to find the maximum sales to get the first case the maximum total profit from 2024 to 2030 is $37008056. Due to multiple uncertainties in climate and market, multiple attributes of the crop will generate uncertainty, consider the fluctuation intervals of uncertain parameters, determine the boxed uncertainty set, and build a robust linear programming model to simulate the uncertainty scenario. The maximum total profit of $35,906,995 is obtained for the years 2024 to 2030 for this scenario.The results show that the robust linear programming model, after considering the uncertainty factors, the maximum total profit, although lower than the deterministic model, is still within the acceptable range, and can effectively improve the robustness and reliability of the planting program and reduce the impact of uncertainty factors on agricultural production, which has a high practical application value.

Keywords: Constrained Approximation, Linear Programming, Robust Linear Programming, Planting Strategies.

1. Introduction

With the development of agricultural production in the direction of intensification and intelligence, the rational use of arable land resources to develop organic planting industry has become an important way to promote the revitalization of rural economy and achieve sustainable development. However, in a mountainous countryside in North China, the complex geographic conditions lead to a variety of types of arable land, covering flat dry land, mountainous land, watered land and various types of greenhouses, etc. At the same time, the climate conditions in the region are variable, which brings a lot of uncertainty to the growth of crops. These factors make planting planning face many challenges, and there is an urgent need for scientific optimization methods to achieve the rational allocation of agricultural resources.

In an existing study, Li Z et al. optimized the planting structure of rice, maize and soybean by NSGA-II algorithm, combining MaxEnt model with improved Hungarian algorithm to achieve spatial layout allocation [1]. A team from the Institute of Agricultural Resources and Agricultural Zoning, Chinese Academy of Agricultural Sciences (CAAS) integrated the MaxEnt model, multi-objective inter-area parameter planning (MOIPP), life-cycle assessment (LCA), and the Dyna-CLUE model for spatial optimization of crop cropping structure, taking into account the uncertainty factor [2]. Wu J et al. combined satellite remote sensing technology and multi-objective optimization algorithms, and carried out a dynamic optimization study of the cropland utilization structure of an area. Combining satellite remote sensing technology and multi-objective optimization algorithm, Wu J et al. carried out dynamic monitoring and optimization of arable land use structure in a region [3]. Zhang Y et al. proposed a spatially coordinated layout optimization method for towns and agriculture based on the PLUS spatial decision-making model [4]. Ni M et al. studied the spatial and temporal patterns and transfer paths of arable land use transition in China, analyzed the influencing factors, and revealed the spatial and temporal change patterns of arable land use transition [5]. Brown L et al. studied the impacts of climate change on the planting patterns of crops in the U.S. Corn Belt, and put forward a planting strategy that adapts to the future climate scenarios [6]. Smith R et al. combined remote sensing and machine learning algorithms to efficiently optimize the spatial distribution of crops in the Murray River, Australia [7]. Zhao W et al. proposed a deep learning-based crop cropping structure optimization model, which improved the rationality and economic efficiency of the cropping structure by analyzing the historical data and the market trend [8]. Gass M P et al. explored the optimization strategy of agricultural planning under the background of climate change by adopting a stochastic planning method [9]. Chen X et al. developed a multi-objective optimization model to achieve the sustainability of agricultural production by optimizing the cropping structure [10]. These studies explored the optimization of arable land use and crop cropping structure from different perspectives, but most of them focused on specific regions or specific crop types, and did not fully consider the impact of climate and market uncertainty on cropping planning.

In this paper, a robust optimization model based on constraint approximation is proposed. First, the model reduces the complexity of the model and improves the solving efficiency by analyzing different plot types and crop planting requirements and simplifying the original constraints. Secondly, the uncertainties are introduced into the model by setting the boxed uncertainty set to make the optimization results more robust and adaptable in the face of uncertainties, taking into account the climate and market uncertainties. Finally, the combination with the linear programming model enables the study to maximize the planting revenue while satisfying all the constraints, providing an effective decision-making tool for the sustainable development of rural economy.

2. Related Theories

2.1. Constrained Approximation

Constraint approximation refers to the operation of simplifying, merging or removing constraints in an optimization problem through mathematical transformations, logical derivations or analysis based on the characteristics of the problem, so as to reduce the number or complexity of constraints under the premise of ensuring the feasibility of the model.

2.2. Linear Programming

Linear programming is used to maximize or minimize an objective function with limited resources. Its model includes an objective function and constraints, and by solving the linear programming problem, an optimal solution for resource allocation can be obtained. The standard form is as follows:

$$\text{maximize } z = \sum_{i=1}^{n} c_i x_i \tag{1}$$

$$\text{S.T} \quad \sum_{i=1}^{n} a_i x_i \leq b, x_i \in \{0,1\}, \text{i}=1,2,...,\text{n} \tag{2}$$

$$x_i \geq 0, \text{i}=1,2,...,\text{n} \tag{3}$$

Where , $c_i$is the coefficients of the objective function,$a_i$is the coefficients of the constraints,$b_i$is the upper bound of the constraints and$x_i$is the vector of decision variables.

2.3. Robust Linear Programming

Robust linear programming is an important branch in the field of robust optimization, aiming at finding a solution that allows the system to satisfy the constraints and minimize the loss of the objective function even in the worst case. Compared with classical linear programming, robust linear programming takes into account the uncertainty of model parameters and is suitable for optimization problems in uncertain environments. Its standard form is as follows:

$$\text{maximize } z = \sum_{i=1}^{n} c_i x_i \tag{4}$$

$$\text{S.T} \quad \sum_{i=1}^{n} a_i x_i \leq b, x_i \in \{0,1\}, \text{i}=1,2,...,\text{n} \tag{5}$$

$$x_i \geq 0, \text{i}=1,2,...,\text{n} \tag{6}$$

where ,$c_i$is the coefficients of the objective function,$a_i$is the coefficients of the constraints,$b_i$is the upper bound of the constraints and$x_i$is the vector of decision variables.In robust linear programming, uncertainty in parametersc_i,a_i,and$b_i$is considered by assuming that they belong to some uncertainty set (e.g., a boxed uncertainty set), and then finding a solution that still satisfies the constraints in the worst case.

3. Experiments

The data used in this paper comes from the dataset provided by Question C of the 2024 Mathematical Modeling Competition. Common planting sites include flat dry land, mountains, watered land, greenhouses and so on, and common plants include rice, wheat, soybeans and so on. Therefore, for the planting requirements of different planting sites and the sales strategy of various types of crops, it is necessary to develop a reasonable sales strategy to maximize the profit of planting. For more than part of the stagnant sales, resulting in waste and more than part of the paper according to the 2023 sales price of 50% of the price reduction in the sale of these two cases.

First of all, the different crops and their planting plot types are summarized and constrained to be simplified, and for the plots that can grow two seasons a year, the equivalent treatment can be treated as two one-season plots a year, as shown in Table 1, D and E type plots can grow two seasons a year, and will be treated as two one-season plots a year.

Table 1: Partial parcel numbering table

Season 1		Season 2	
tectonic plate	serial number	tectonic plate	serial number
D1	27	D1	55
D2	28	D2	56
D3	29	D3	57
D4	30	D4	58
D5	31	D5	59
D6	32	D6	60
D7	33	D7	61
D8	34	D8	62
E1	35	E1	63
E2	36	E2	64

After the equivalent treatment, according to the distribution of crops, the flat dry land, mountainous and hillside land and watered land, ordinary greenhouses and intelligent greenhouses are classified into three major categories, such as Fig. 1, and are processed in chunks, and for the analysis of the planting strategy's on each chunk, the constraints can be reduced accordingly, for example, the fungus is located in the sub-chunk 3 can be exempted from the constraints of the legumes that must be planted within 3 years, because the area planted in the sub-chunk 3 is contained in chunk 1, only the planting of chunk 1 needs to be considered to satisfy the constraint that beans must be planted within 3 years, and chunk 3 is also not subject to the constraint of rice planting.

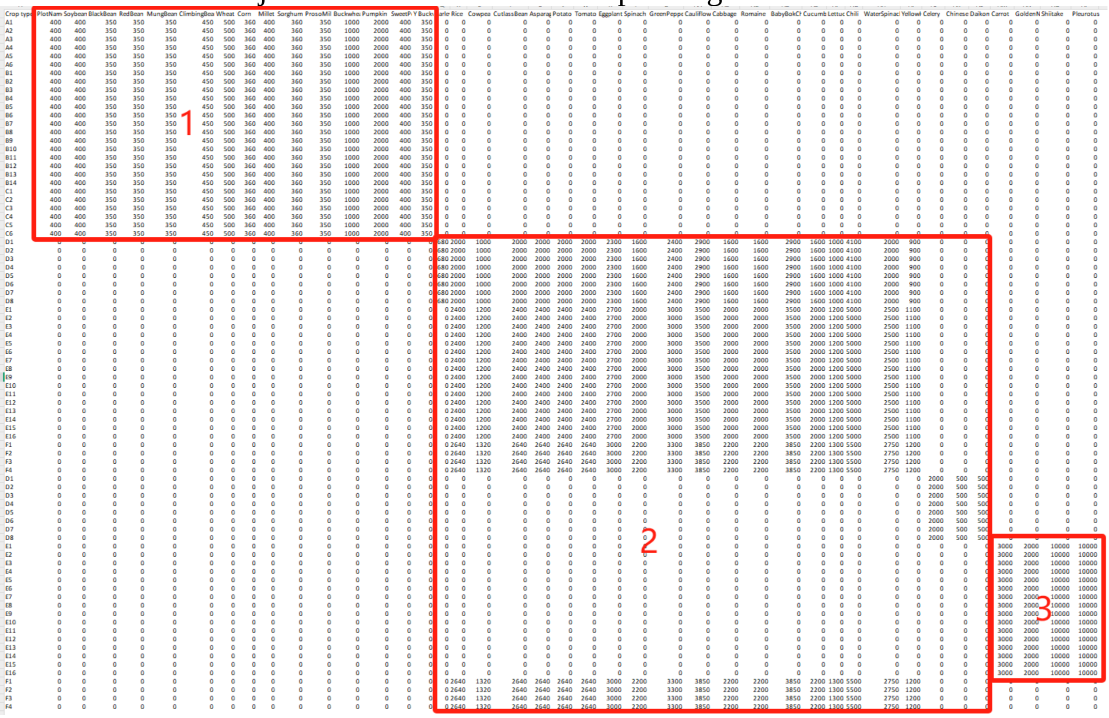

Figure 1: Map of topographic subdivisions

Set the decision variable as the planting area of the plant, as the planting per acre profit of a certain plant in a certain quarter, for different subsections, consider and select the corresponding constraints: at least once in three years to plant legumes, the total area of the crop, the total production of each crop is less than the expected sales, two years do not plant the same crop and other constraints, and then introduce the linear programming model, bring in the objective function, more than the portion of the stagnant sales, resulting in a wasteful The situation through MATLAB to establish a linear programming model to maximize the total return, to solve the maximization of total return and the optimal planting program. After the algorithm solving and analyzing this paper to get the maximum planting profit and planting strategy between 2024-2030.

For the three types of chunks, respectively, to establish a linear programming model to solve the linear programming problem, we need to determine the model of the decision variables, objective function and constraints, for different situations, based on the choice of constraints and the establishment of different objective functions.

(1)Decision variables

$$b_j=\sum_{i=1}^{42} x_{i,j,t} \tag{7}$$

where b_j is the total area of the plot, $x_{i,j,t}$ is the area planted to each crop per year, i is the plot area, j is the crop number and t is the year.

(2)Restrictive condition

Condition a::

j=12 i=27,28···82

j=16 i=1,2···26 ; 35,36···82

j=17,18···34 i=1,2···26; 55,56···72

j=35,36,37 i=1,2···51; 63,64···82

j=38,39···41 i=1,2···63; 79,80···82

where j is the crop number and i is the plot where crop j can be grown.

Condition b:

$$\text{if } x_{ijt}\neq 0 \quad \text{then } x_{ijt+1}=0 \tag{8}$$

where $x_{i,j,t}$ is the annual acreage of each crop, and when that crop has been planted in one year, it is not planted in the second year.

Condition c:

When m=1,2,3,4,5

$$\sum_{t=m}^{m+2}\sum_{j=1}^{5} x_{i_{j,t}} \neq 0 \qquad i=1,2\cdots 26 \tag{9}$$

$$\sum_{t=m}^{m+2}\sum_{j=17}^{19} x_{i_{j,t}} \neq 0 \qquad i=27,28\cdots 50 \tag{10}$$

$$\sum_{t=m}^{m+2}\sum_{j=17}^{19} (x_{i_{j,t}}+x_{i+28,j,t}) \qquad i=51,52\cdots 54 \tag{11}$$

where Equation 10 and Equation 11 indicate that the area of the legume crops numbered 1 to 5 and 17 to 19 to be planted in a three year period is to be greater than 0. Equation 12 equation then indicates the total area planted in two seasons in a three year period where $x_{i,j,t}$ indicates the area of the legume planted in the first season, and $x_{i+28,j,t}$ indicates the area of the legume planted in the second season. Condition d:

$$\sum_{m}^{m+2}\sum_{j=1}^{5} x_{i_{j,t}} \neq 0 \quad i=1,2\cdots 26 \tag{12}$$

where $x_{i,j,t}$ is the area planted per year for each crop, and numbered 1 to 5 are the total area planted in three years for the legume crops.

Condition e:

$$\max\{sum \sum_{t=2}^{8} p_{i,j} \, [u+\min(A_{i,j}x_t\text{-}u,0)]\} \tag{13}$$

where the difference between the total production of the jth crop in the middle of year t and the total production in the year 2023 and the minimum value is taken in comparison with zero. The minimum value obtained plus the total production in 2023 is multiplied by the price of the jth crop in the ith plot, and then by accumulation, the maximum value of the objective function can be obtained.

(3)Objective function:

$$\max\{\sum\sum_{t=2}^{8} p_{i,j} \, [u+\min\left(A_{i,j}x_t\text{-}u,0\right)]\} \tag{14}$$

where the difference between the total production of the jth crop in the year t and the total production in the year 2023 and is minimized in comparison with 0. And calculate the total sales of the jth crop in year t and the total production of that year, will be obtained by adding the two results and the jth crop in the price of the ith land multiplied, and then through the cumulative, can be derived from the maximum value of the objective function.

Due to the multiple uncertainties of climate and market, the expected sales volume, acre yield, planting cost and sales price of crops will generate uncertainty, this paper considers the fluctuation interval of uncertain parameters and determines the box-type uncertainty set, and establishes a robust linear programming model to simulate the uncertainty scenario.

(1) Uncertainty description

The key to robust optimization is how uncertainty is described. It is common practice to represent and as containing uncertainty in the form.

$$L=L_0+\Delta L,\ d=d_0+\Delta d \tag{15}$$

where L and d are nominal values, ΔL and Δd are uncertainty components and they belong to some uncertainty set z. Common uncertainty sets are:

Boxed Uncertainty Set:Each element is perturbed in a definite interval, i.e.

$$U=\left\{\Delta L,\Delta d:|\Delta L_{i,j}|\leq\delta_{L_{i,j}},|\Delta d_i|\leq\delta_{d_i}\right\} \tag{16}$$

The uncertainties involved in this paper are boxed uncertainty sets

(2) Forms of robust optimization problems

After introducing uncertainty into the standard linear programming problem, the robust linear programming model becomes.

$$\min_x C^T x \tag{17}$$

$$subjectto(L_0+\Delta L)x\leq d_0+\Delta b \quad \forall(\Delta L,\Delta d)\in U \tag{18}$$

where U is the uncertainty set,denoting all possible ranges of ΔL and Δd values.

(3) Deterministic equivalence

Under the assumption of certain uncertainty sets (e.g., boxed uncertainty sets or polyhedral uncertainty sets), the robust optimization problem can be transformed into a deterministic optimization problem. In the case of boxed uncertainty sets, for example, their robust constraints can be transformed into a stronger set of linear inequality constraints, thus transforming the robust optimization problem into a standard linear programming problem.

Assuming that ΔL and Δd belong to the boxed uncertainty set, the robust linear programming constraint.

$$(L_0+\Delta L)x\leq d_0+\Delta d, \qquad \forall\Delta L,\Delta d\in U \tag{19}$$

equivalence is (math.)

$$L_0x+\max_{\Delta L\in U}\Delta Lx\leq d_0+\min_{\Delta d\in U}\Delta d \tag{20}$$

This transformation allows the robust problem to be solved by a classical linear programming solver.

4. Results

Combining the objective function as well as the constraint equations, this paper classifies flat dry land, mountain and hillside land with watered land, ordinary greenhouses and smart greenhouses into three broad categories based on the distribution of crops, and determines the parcels included in the different subplots as shown in Table 2:

Table 2: Table of parcels included in the subdivision

Block numbering	Includes plots
Subchapter 1	A1~A6,B1~B14,C1~C6
Subchapter 2	D1~D8,E1~E16,F1~F4
Subchapter 3	E1~E16

Which the same subplot on the plot has a high degree of connectivity, the connection is reflected in the crop rotation, heavy cropping, and land area limitations, that is, the same subplot on the plot in the constraints have the consistency of the constraints, through the different plots of the reasonable subplot, we choose to target the selection of constraints, reducing the constraints and the model to solve the complexity of the problem.

Under the consideration of more than part of the stagnant sales and cause waste, through the constraints of approximation and linear programming to get the flat optimal planting program under the sales of 2024 ∽ 2030 each year is shown in Table 3.

Table 3: 7 years of sales under stagnation and wastage

particular year	margins
2024	5578006
2025	4980073
2026	5167801
2027	5710070
2028	4435608
2029	4600980
2030	6335518

Among them, the sales between different years have a high degree of correlation, and do not show a year-by-year upward or downward trend, but based on a certain range of fluctuations, reflecting the model has a strong stability, and it is not easy to fall into the local optimum when solving, which is conducive to our development of optimal planting strategy.

Under the consideration of more than part of the stagnant sales and waste, and the introduction of uncertainty, through the constraints of approximation and robust linear programming to obtain the optimal planting program under the optimal planting program in 2024 ∽ 2030 each year's sales are shown in Table 4.

Table 4: 7 Years of Sales with Stalled Sales and Resulting Discounts

particular year	margins
2024	4998006
2025	5180370
2026	5393700
2027	4953091
2028	4797088
2029	5006940
2030	5577800

Comparing the data in Table 3 with those in Table 2, it can be seen that with the introduction of robust optimization to resist uncertainties, the model tends to be more conservative in its results, which is conducive to safeguarding the benefits of workers and improving the ability to resist risks.

The above experimental results show that the model proposed in this paper also maximizes the overall economic benefits under the constraints of planting area, land type and scientific planting requirements at the same time. Under the stagflation scenario, the model tends to increase the planting area allocation of high-demand crops and reduce resource waste.

5. Conclusions

In this paper, a crop planting strategy under robust linear planning based on constraint approximation is proposed, aiming at combining the rural reality, scientifically allocating crop and plot resources, formulating the optimal planting plan, and maximizing the planting income in the next 7 years. The algorithm combines matrix analysis and optimal planning, reduces the model solution complexity through constraint parsimony, introduces robust optimization to adapt to uncertain environments, reduces potential losses, and screens out the optimal or near-optimal solutions under uncertain conditions, which enhances the interpretability and reliability of the model. However, there is still room for improvement of the model, and the robust linear programming conditions are more strict, and the results may be too conservative.

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