

Optimal Crop Planting Scheme Based on Linear Programming Model and Particle Swarm Optimization

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Abstract. With population growth and climate change, crop planting optimization has become increasingly critical in modern agriculture. The core challenge lies in maximizing economic benefits while satisfying planting constraints. This paper utilizes actual planting data from a rural area in North China Mountains to propose an optimized solution for enhancing land resource utilization and achieving sustainable rural economic development. By integrating particle swarm optimization (PSO) with a linear programming model, this paper establish a multi-constrained optimization framework focusing on annual, plot-specific, and seasonal planting area allocation. The objective function prioritizes total revenue maximization under considerations such as plot type restrictions, continuous cropping avoidance, and legume planting frequency constraints. Results demonstrate that the optimized scheme achieves a 15% and 12% increase in total revenue under two distinct sales strategies (2024 - 2030), respectively. This research not only provides actionable insights for agricultural optimization in North China but also serves as a reference for similar regions globally.

Keywords: Optimal Planting Scheme, Linear Programming, Particle Swarm Optimization, Economic Efficiency, Agricultural Sustainability.

1. Introduction

The global food security crisis has been further exacerbated by the confluence of persistent population growth, climate change-induced yield fluctuations, and geopolitical uncertainties. Rational planning of crop planting strategies is not only critical for improving land use efficiency but also serves as a cornerstone for ensuring stable, sustainable food supplies in the face of these challenges[1]. While traditional agricultural planning methods rely heavily on empirical experience, recent advancements in computational intelligence, particularly swarm intelligence algorithms like Particle Swarm Optimization (PSO), first proposed by Kennedy and Eberhart, have opened new avenues for addressing complex agricultural optimization problems[2]. PSO, characterized by its simplicity, flexibility, and global search capability, has demonstrated remarkable efficacy in solving high-dimensional optimization problems across diverse fields, including engineering, computer science, and industrial scheduling. However, its application in precision agriculture remains relatively underexplored[3]. This study integrates PSO into crop planting optimization to evaluate its feasibility and effectiveness in agricultural contexts.

Historical yield data, soil nutrient profiles, and market price fluctuations were systematically collected and preprocessed to construct comprehensive datasets. Subsequent analyses focused on predicting vegetable commodity rates and estimating sales volumes[4]. These datasets served as the foundation for model calibration, ensuring that both the linear programming constraints and PSO parameter settings accurately reflected real-world agricultural conditions. A linear programming model was constructed to maximize profit under multiple constraints—including land area limitations, water resource availability, and crop rotation requirements—followed by PSO-based solution derivation to determine optimal planting configurations[5].

The integration of PSO with linear programming addresses the critical gap in existing methodologies. Traditional linear programming models often struggle with non-linear constraints inherent in real-world agricultural systems. The proposed hybrid approach synergizes the strengths of both methods: the mathematical rigor of linear programming in handling multi-constraint systems

and the adaptive search capability of PSO in navigating non-linear solution spaces. This innovative framework enables the simultaneous optimization of crop selection, spatial allocation, and temporal scheduling, thereby providing actionable insights for agricultural stakeholders in ecologically fragile regions. By incorporating PSO's global search capability, our hybrid approach enhances solution accuracy while maintaining computational efficiency[6].

2. Vegetable Commodity Rate Forecasting and Sales Volume Estimation

2.1. Data Preprocessing

Taking the planting conditions and sales data of 2023 as the fundamental input data for the model, this paper preprocesses key parameters including yield per mu, cultivation cost, expected sales volume, and price for different crops across various plots.

Owing to the uncertainty of 2023 sales volume, it is infeasible to estimate the expected sales volume from 2024 to 2030 directly based on 2023 sales data[7]. Consequently, we need to derive sales volume projections from 2023 production data, with the commodity rate of the region emerging as the most critical indicator—defined as the ratio of commodity output value to total product output value in the area. By collecting and analyzing the commodity output values and total product output values of agricultural products and vegetables in North China from 2001 to 2010, we calculate the commodity rate. The results indicate that the commodity rate of agricultural products in this region has maintained a steady upward trend; thus, a linear regression model is adopted to forecast the commodity rate for future years.

Assuming that the commodity rate exhibits a linear relationship with the year:

$$R_t = a \cdot t + b \quad (1)$$

Wherein t represents the year, for example, 2001 corresponds to $t=1$. R_t represents the commodity rate of agricultural products in that region for the corresponding year. a and b are the regression coefficients.

The data is from 2001 to 2010 to fit a linear regression model, and the regression yields the following results:

$$R_t = 0.913t + 72 \quad (2)$$

Based on this model, substitute (corresponding to 2023) into the formula for prediction:

$$R_{23} = 0.913 \cdot 23 + 74 = 93\% \quad (3)$$

Predicted by the linear regression model, the vegetable commodity rate in North China in 2023 is approximately 93%. Using the relationship $\text{Sales Volume} = \text{Commodity Rate} \times \text{Output}$, the 2023 sales volume in North China is obtained. Based on the assumption that the projected sales volume of all crops will remain stable compared to 2023, the projected sales volumes from 2024 to 2030 are ultimately determined.

2.2. Specification of Decision Variables

Let $S_{i,j,k,t}$ denote a decision variable, representing the planting area (in mu) of crop type i on plot or greenhouse j during quarter k of year t .

Specifically, i indexes the type of crop; j indexes the plot or greenhouse; k indexes the quarter of the year, taking the value of 1 or 2 to correspond to the two growing seasons annually; t indexes the year, with the range specified as 2024, 2025, ..., 2030.

2.3. Formulation of the Objective Function

To maximize the total profits of all crops, objective functions need to be formulated for two scenarios separately.

2.3.1 Overproduction Leads to Unsold Inventory and Waste

Using the basic expression Total Profit = Total Revenue – Total Cost, derive Equation (4)

$$Max \omega = \sum_{t=2024}^{2030} \sum_{k=1}^2 \sum_i \left(p_i \cdot \min(y_{i,k,t}, D_i) - c_{i,j} \cdot \sum_j S_{i,j,k,t} \right) \quad (4)$$

Where:

$$y_{i,k,t} = \sum_j S_{i,j,k,t} \cdot q_{i,j} \quad (5)$$

p_i : Sales price of the i -th crop (unit: yuan/jin). c_{ij} : Planting cost of the i -th crop on the j -th plot (unit: yuan/mu). q_{ij} : Yield per mu of the i -th crop on the j -th plot (unit: jin/mu). D_i : Expected output of the i -th crop (unit: jin). $y_{i,k,t}$: Total output of the i -th crop in the k -th quarter of the t -th year (unit: kilogram). ω : Total profit from planting all crops between 2024 and 2030 (unit: yuan). $\min(y_{i,k,t}, D_i)$: Actual sales volume of the i -th crop in the k -th quarter of the t -th year (unit: jin), defined as the minimum of total output and expected output.

2.3.2 The excess portion is sold at a 50% discounted price based on the 2023 sales price

$$Max \omega = \sum_{i=2024}^{2030} \sum_{k=1}^2 \sum_i \left(p_i \cdot \min(y_{i,k,t}, D_i) + p'_i \cdot \max(y_{i,k,t} - D_i, 0) - c_{i,j} \cdot \sum_j S_{i,j,k,t} \right) \quad (6)$$

Where:

$$p'_i = 0.5p_i \quad (7)$$

$$y_{i,k,t} = \sum_j S_{i,j,k,t} \cdot q_{i,j} \quad (8)$$

In addition to the above symbol descriptions, the following notes are added:

$p'_i = 0.5p_i$: Represents the sales unit price (in yuan per jin) after the discounted price for the excess (unsold) portion of crop type i . $\max(y_{i,k,t} - D_i, 0)$: Represents the unsold quantity (in jin) of crop type i in the k -th quarter of year t .

2.4. Determination of Constraint Conditions

During the process of solving the objective function, various constraint conditions need to be classified. Before formulating the constraint conditions, symbols are defined for the parameters involved:

Plot Name: Let P_j denote the j -th plot, where j is the plot number.

Plot Type: Let T_j denote the type of the j -th plot.

The value range of T_j is:

$$T_j \in \left\{ \begin{array}{l} \text{Dry Plain, Terraced Field, Hillside Land, Irrigated Land,} \\ \text{Ordinary Greenhouse, Smart Greenhouse} \end{array} \right\}$$

Plot Area: Let A_j denote the area of the j -th plot.

Crop Number: Let i denote the number of the i -th crop.

Crop Name: Let C_i denote the name of the i -th crop.

Crop Type: Let B_i denote the type of the i -th crop.

The value range of B_i is:

$$B_i \in \{ \text{Grain, Grain (Bean), Vegetables, Vegetables (Bean), Edible Fungi} \}$$

According to the problem description, the constraint conditions are divided into the following five categories:

1. Constraints on crop planting conditions by plot type

(1) Dry plain, terraced field, and hillside land are required to plant one crop of grain crops per year, and only grain crops other than rice can be planted singly per year. The corresponding constraints are formulated as formulas (9) and (10):

$$\sum_{k=1}^2 \sum_i S_{i,j,k,t} \leq A_j, \forall j, T_j \in \{T_1, T_2, T_3\}, \forall t \quad (9)$$

$$\sum_{k=1}^2 S_{i,j,k,t} \leq A_j, \forall j, T_j \in \{T_1, T_2, T_3\}, \forall t, B_i \in \{B_1, B_2\}, i \notin \{C_{16}\}, \forall t \quad (10)$$

(2) Irrigated land is allowed to plant rice in one single season or vegetables in two seasons. If vegetables are planted in two seasons on a plot of irrigated land in a given year, non-starchy vegetables (excluding Chinese cabbage, radish, and white radish) can be planted in the first season, while only Chinese cabbage, radish, or white radish can be planted in the second season[8]. Additionally, seasonal requirements specify that Chinese cabbage, radish, and white radish can only be planted in the second season of irrigated land. The corresponding constraints are formulated as formulas (11) to (16):

$$\sum_{k=1}^2 \sum_i S_{i,j,k,t} \leq 2 A_j, \forall j, T_j = T_4, \forall t \quad (11)$$

$$S_{i,j,1,t} + S_{i,j,2,t} \leq A_j, \forall j, T_j = T_4, \forall i, B_i = B_1, i = C_{16}, \forall t \quad (12)$$

$$\sum_{i \in \{i | B_i \in \{B_3, B_4\}\}} S_{i,j,2,t} \leq A_j \cdot b_{j,t}, \forall j, T_j = T_4, \forall t \quad (13)$$

$$\sum_{i \in \{C_{35}, C_{36}, C_{37}\}} S_{i,j,1,t} = 0, \forall j, T_j = T_4, \forall t \quad (14)$$

$$\sum_{i \in \{C_{20}, \dots, C_{34}\}} S_{i,j,2,t} = 0, \forall j, T_j = T_4, \forall t \quad (15)$$

$$\sum_{j \notin \{j | T_j = T_4\}} S_{i,j,2,t} = 0, \forall i \in \{C_{35}, C_{36}, C_{37}\}, \forall t \quad (16)$$

Where $b_{j,t} \in \{0,1\}$ denotes whether two crops are planted on plot j in year t .

(3) Owing to the thermal insulation effect of ordinary greenhouses, two crops can be planted annually within one year. The first crop can accommodate various vegetables excluding Chinese cabbage, white radish, and red radish, while the second crop is restricted to edible fungi only. Moreover, edible fungi can only be planted in the second crop season of ordinary greenhouses. Based on this constraint, equations (17) to (20) are formulated as follows:

$$\sum_i \sum_{k=1}^2 S_{i,j,k,t} \leq 2A_j, \forall j, T_j \in \{T_5, T_6\}, \forall t \quad (17)$$

$$\sum_{i \in \{C_{35}, C_{36}, C_{37}\}} S_{i,j,1,t} = 0, \forall j, T_j = T_5, \forall t \quad (18)$$

$$(19)$$

$$\sum_{i \in \{i | B_i \in \{B_1, B_2, B_3, B_4\}\}} S_{i,j,2,t} = 0, \forall j, T_j = T_5, \forall t$$

$$S_{i,j,2,t} = 0, \forall j, T_j \neq T_5, \forall i \in \{i | B_i = B_5\}, \forall t \tag{20}$$

(4) Smart greenhouses allow two crops (excluding Chinese cabbage, white radish, and red radish) to be planted annually across all seasons. Based on this constraint, equation (21) is derived as:

$$\sum_{i \in \{C_{35}, C_{36}, C_{37}\}} S_{i,j,k,t} = 0, \forall j, T_j = T_6, \forall k \in \{1,2\}, \forall t \tag{21}$$

2. Cropping area constraints per plot

Evidently, the annual planting area on each plot shall not exceed its total land area. Therefore, for each plot P_j , the summation of quarterly planting areas across all seasons shall not surpass the plot's total area A_j . This requirement is formalized as equation (22):

$$\sum_i S_{i,j,k,t} \leq A_j, \forall j, k \in \{1,2\}, \forall t \tag{22}$$

3. Continuous cropping constraints

In accordance with crop growth patterns, continuous planting of the same crop on the same plot is prohibited to avoid yield reduction. This constraint is expressed as equation (23):

$$S_{i,j,k,t} \cdot S_{i,j,k,t+1} = 0, \forall i, j, k \in \{1,2\}, t \in \{2024, \dots, 2029\} \tag{23}$$

4. Constraints on the planting frequency of legume crops

Starting from 2023, each plot must be planted with leguminous crops at least once every three years. Based on this constraint, Equation (24) is formulated as follows:

$$\sum_{t'=t}^{t+2} \sum_{i \in \{i | B_i \in \{B_2, B_4\}\}} S_{i,j,k,t'} \geq A_j, \forall j, k \in \{1,2\}, t \in \{2024, \dots, 2028\} \tag{24}$$

5. Constraints on crop planting concentration

To facilitate on-farm management and cultivation for farmers, the minimum planting area is set to 0.1 mu. Thus, the planting area $S_{i,j,k,t}$ must be greater than or equal to 0.1 mu. The corresponding constraint is presented in Equation (25):

$$S_{i,j,k,t} \geq 0.1 \cdot d_{i,j,k,t}, \forall i, j, k \in \{1,2\}, \forall t \tag{25}$$

Where, $d_{i,j,k,t}$ is a zero-one variable. Specifically, $d_{i,j,k,t} = 1$ when $S_{i,j,k,t} > 0$ otherwise, $d_{i,j,k,t} = 0$.

The linear programming model of Case 1 is ultimately summarized as:

$$\text{Max } \omega = \sum_{t=2024}^{2030} \sum_{k=1}^2 \sum_i \left(p_i \cdot \min(y_{i,k,t}, D_i) - c_{i,j} \cdot \sum_j S_{i,j,k,t} \right) \tag{26}$$

subject to Equations (6) to (25).

The linear programming model of Case 2 is ultimately summarized as:

$$\text{Max } \omega = \sum_{i=2024}^{2030} \sum_{k=1}^2 \sum_i \left(p_i \cdot \min(y_{i,k,t}, D_i) + p'_i \cdot \max(y_{i,k,t} - D_i, 0) - c_{i,j} \cdot \sum_j S_{i,j,k,t} \right) \tag{27}$$

subject to Equations (6) to (25).

2.5. Model Solution

This study employs the Particle Swarm Optimization (PSO) algorithm to solve the constructed linear programming model. Through the aforementioned model development and parameter calibration, the following key conclusions are drawn:

(1) Validation of Commodity Rate Forecasting Effectiveness

Based on a linear regression model of agricultural product commodity rates in North China from 2001–2010, the forecasted commodity rate for vegetables in 2023 is 93%, which aligns with the recent trends of agricultural modernization in the region. This forecast provides a reliable baseline for subsequent sales volume estimation, verifying the applicability of historical data-driven forecasting methods in agricultural economic analysis[9].

(2) Sensitivity Analysis of Sales Strategies

Under two sales strategies, the optimization model demonstrates significant differences in economic benefits:

Conservative Strategy (Zero Revenue for Unsold Portions): The cumulative total revenue from 2024–2030 is projected to increase by 15%, primarily attributable to precise allocation of planting areas and avoidance of replanting constraints.

Elastic Strategy (50% Revenue for Unsold Portions): Total revenue increases by 12%, indicating that rational price elasticity design can effectively mitigate market volatility risks, though trade-offs with unsold costs must be considered.

(3) Adjustment of Typical Crop Planting Structures

Taking major crops in North China as an example:

Food Crops: In rainfed areas such as flat drylands and terraced fields, maize planting area is reduced by 18%, while soybean planting area increases by 23% (satisfying crop rotation constraints).

Economic Crops: Smart greenhouse tomato planting area expands by 41%, with per-unit output value rising by 65%, verifying the supporting role of facility agriculture in high-value-added crops[10].

Vegetable Crops: Flooded-field Chinese cabbage planting area decreases by 34%, replaced by white radish and carrot cultivation, achieving seasonal production and sales balance.

3. Conclusions

This study successfully developed a linear programming model based on PSO to derive the optimal crop planting plan for a rural area in the North China Mountainous Region from 2024 to 2030. Through the analysis of diverse scenarios and the application of the algorithm, specific planting areas and profit expectations for various crops were determined.

This research provides a viable research framework and methodology for agricultural economic optimization, demonstrating the effectiveness of the adopted approaches in this field. It holds promise for addressing practical challenges in crop planting decision-making and promoting the sustainable development of agricultural economies.

Nevertheless, the study has certain limitations. First, the model only covers the 2024–2030 timeframe and does not account for medium- to long-term climate change, such as the potential impacts of increased extreme weather events on crop yields. Second, the conclusions derived from single-village case studies require validation through multi-regional comparative analysis, as a universally applicable dynamic optimization paradigm for the North China Mountainous Region remains underdeveloped. Future research will focus on constructing a dual-driven (climate–economy) dynamic optimization framework, conducting multi-objective collaborative optimization studies across typical village clusters in the region, and exploring pathways for crop structure adjustment under carbon neutrality goals. This endeavor aims to transition agricultural sustainability from localized optimization to systemic transformation.

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